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Experiments in nonlinear dynamics using control-based continuation: Tracking stable and unstable response curves

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Summary. We show how to implement control-based continuation in a nonlinear experiment using existing and freely available software. We demonstrate that it is possible to track the complete frequency response, including the unstable branches, for a harmonically forced impact oscillator.

Introduction

We show how to perform experimental bifurcation analysis for nonlinear dynamical systems using control based continuation, in particular tracking complete frequency response curves including their unstable parts. Nonlinear dynamical systems are difficult to deal with experimentally because of possible coexistence of stable and unstable equilibrium states, super/sub harmonic resonances, quasi-periodic and chaotic behaviour. Many of the well established experimental methods use estimation and identification techniques that are based on the assumption that the system under test is linear or close to linear. Applying such techniques to strongly nonlinear systems can lead to incorrect measurements and hence wrong model-assumptions, poor designs and failure of mechanical components. Experimental techniques for stabilizing unstable periodic orbits (UPOs) in chaotic systems, such as Delayed Feedback Control and OGY-control, are emerging (see [1] for an overview). For nonlinear mechanical systems with periodic or quasi-periodic behaviour, the parameter-sweep remains the only widely used counter-part to linear methods such as experimental modal analysis. Unfortunately, this method does not provide any information about unstable equilibrium states and can only handle co-existence of equilibrium states to a certain degree. The newly developed control-based continuation requires the constitution of a non-invasive real-time control and the use of a predictor-corrector type path following algorithm, but in turn the method can provide information about how both stable and unstable equilibrium states change when system parameters are varied. Furthermore, the stability can be determined and in some cases the instability can be quantified in terms of finite-time Lyapunov Exponents [2]. The method works for linear, weakly nonlinear and strongly nonlinear systems and can handle multiple co-existing equilibrium states, quasi-periodic behaviour and the occurrence of bifurcations. In the following we give an overview of how one can apply this method in experiments using freely available software.

Experimental test rig

The experimental test-rig is shown in Figure 1a and b. It comprises a harmonically forced impact oscillator controlled by electromagnetic actuators. The harmonic excitation F_s is created by a electromagnetic shaker attached to the base and a control force F_m can be exerted directly on the impactor mass using the electromagnetic actuators. Data acquisition and the generation of control and forcing signals are done using a dSpace (`http://www.dspace.com`) DS1104 real-time control board.

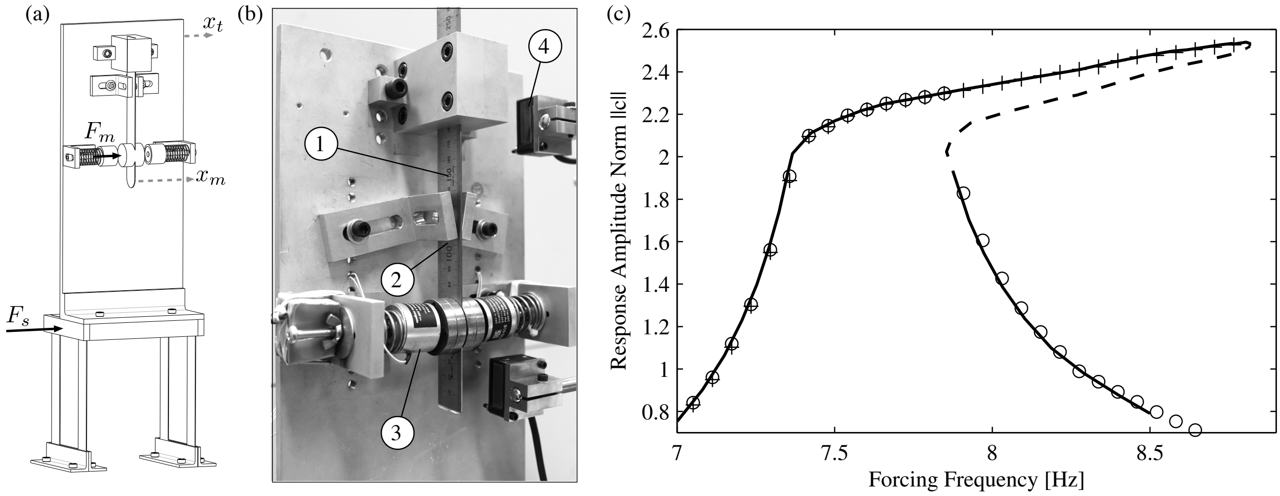

Figure 1: The experimental test-rig and results obtained using different experimental methods. (a) Illustration of the harmonically forced impact oscillator controlled by electromagnetic actuators. (b) Close-up picture of the impactor system: (1) Flexible impactor with tip mass; (2) mechanical stops causing impact and a hardening spring nonlinearity when vibration amplitudes exceeds the gap-size; (3) electromagnetic actuators used to create a non-invasive control force; (4) laser displacement sensors. (c) Comparison of an experimental frequency response obtained by frequency sweep and control-based continuation. Sweep is denoted by (+) for increasing frequency and (o) for decreasing. Continuation is denoted by (—) for stable and (- - -) for unstable equilibrium branch.

Implementing control-based continuation in an experiment

Control-based continuation was introduced in [3] and has been applied to different experiments in [4, 5, 6, 7, 8]. It employs a path following algorithm to track response curves, while the equilibrium states are stabilized by a non-invasive control. Some pre-requisites are necessary for control-based continuation: 1) It must be possible to vary the parameters of interest (e.g. forcing frequency and amplitude) smoothly. 2) A zero-problem and an appropriate interface must be set up, so that the continuation algorithm can evaluate the experiment. For time periodic equilibrium-states, such a zero-problem can be formulated as:

$$F(c,\mu;N) := \mathcal{F}_Q\left(Y(\mu,N,\mathcal{F}_\infty^{-1}(c))\right) - c = 0, \tag{1}$$

where Y is the measurement of the controlled experiment after transients have settled, N is the number of sampled points, μ are parameters, Q is the number of modes used in the Fourier transformation $\mathcal{F}_Q$ and c is the reference state expressed terms of its Fourier modes. The continuation algorithm makes a prediction step in paramteres μ and reference state c based on an experimentally estimated Jacobian. This prediction is realized in the experiment by the control. Subsequently, a corrector algorithm corrects the reference state c until the reference state and measurements match. 3) A non-invasive control must be realised. By non-invasive control we mean a control that is only active when the system is away from an equilibrium state of the underlying un-controlled system. This also means that if the control-actuators are not already a part of the system, they must not add any inertia, stiffness, damping or extra degrees of freedom. In some cases the control can be overlaid the external excitation. A non-invasive control can be constituted as a PD control, G, with appropriately chosen gains K_p and K_d. The control signal $u_c(t)$ is thus expressed as

$$u_c(t) = G(x(t), y(t)) := K_p(x(t) - y(t)) + K_d(\dot{x}(t) - \dot{y}(t)), \tag{2}$$

where $y(t)$ is the measurement of the state of the controlled system at time t and $x(t) = \mathcal{F}_\infty^{-1}(c)$ is the reference trajectory produced by the continuation. When the correction converges $x(t) - y(t) \approx 0$ and $u_c \to 0$, meaning that one does in fact measure the local dynamics of the underlying un-controlled system. As presented in [8], the control gains K_p and K_d can be experimentally tuned by a performing a series of sweeps determining their ability to non-invasively stabilize stable and unstable periodic states under influence of external perturbations. The control gains generally depend on the nonlinearity of the system, but for many cases it is possible to determine a single set of gains that enables the full bifurcation diagram to be tracked. A Matlab/Simulink software toolbox called Continex (Continuation in experiments), which generates the non-invasive control signal, creates and evaluates a zero-problem and handles communication between an experiment and a numerical continuation code, has been developed and is freely available together with the continuation code COCO [9].

Conclusions

Figure 1c shows a frequency response obtained using both conventional parameter sweep and the control-based continuation method. The method is seen to be able to handle multiple co-existing equilibrium states, trace both the stable and unstable equilibrium states and determine their stability. The time needed for obtaining the experimental results is of the same order as the time needed for a parameter-sweep. Several measurements are made and statistically weighted for each accepted point along the response curve, and since the continuation algorithm only accepts a state as an equilibrium when the residuum (1) is sufficiently small, the quality of the measured data is ensured. A non-invasive control is necessary for the method to work, but in some cases the control force can be overlaid on the external excitation. Furthermore, many advanced electro-mechanical components, such as rotors supported by electromagnetic bearings, already include the necessary hardware. Control-based continuation is still under development, but it can be considered a suitable alternative to conventional parameter-sweeps. It will work in many situations where the parameter-sweep fails and it can provide valuable information about the unstable equilibrium states. The Matlab/Simulink Continex software toolbox which can be downloaded from [9] makes it easy to set up control-based continuation experiments.

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